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Different ways to consider heterogeneity in quase-fragile materials using a version of lattice model

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Abstract

The simulation of the damage process in quasi-fragile materials may be classified in two large groups, those based on Continuum Mechanics, i.e. the so-called Classical approach and the Statistical Models approach. In the former, plasticity theories are extended to study the damage process, leading to procedures that encounter serious difficulties when dealing with quasi fragile materials, in which scale effects, anisotropic damage and associative behavior among defects are likely to occur. In the present work a version of lattice method is used to simulate the behavior of cubic models submitted to uniaxial tension and compression load. Different levels of ductility were considered in the models using several setting of material properties in their definition. Over the cubic models thus implemented, two source of heterogeneities were considered: Introducing parameters that govern the uniaxial constitutive law of the bars as random fields and introducing imperfection in the coordinates of the lattice mesh. Results in terms on the global strain and stress, energy balance and final configuration are presented.

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Keywords: Heterogeneous materials, Lattice Model, Statistical Approach .

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1 Introduction

According to Krajcinovic (1996), the methods proposed to predict the damage process in quasi-fragile materials may be classified in two large groups, those based on Continuum Mechanics, *i.e.* the so-called classical approach and the Statistical Models approach. In the former, Plasticity Theories are extended to study the damage process, leading to procedures that encounter serious difficulties when dealing with quasi fragile materials, in which scale effects, anisotropic damage and associative behavior among defects are likely to occur.

Examples of the Continuum Mechanics approach are the classical model due to Ottosen (1975) and the recent contribution of Crawford *et al.* (2012), who proposed a procedure implemented in the commercial finite element package *LS-Dyna* (Hallquist, 2007) to model damage in quasi-fragile materials.

On the other hand, in the second group (Statistical Models), the versatility of the finite element approach is lost, but in compensation problems without the hard hypotheses to consider that the the microfissuration have a uniform spatial distribution and that the stress field in the proximities of a microfissure independ of the position of their neighbors. Examples of the second approach are provided by Li and Liu (2002), who review resort to discrete models consisting of particles in a mesh free distribution. This method was incorporated to the 2012 version of the commercial Finite element program *Abaqus* (2012) confirming a perceptible tendency of the scientific community to resort to so-called statistical methods, as an appealing alternative to solve problems in which discontinuities appear during the damage process. Lattice Models, of which the formulation used in the present work, called as truss like Discrete Element Method (DEM) is a particular case, belong to this group. Basically, the solid is modeled by means of an array of uniaxial elements, which interconnect nodal masses with two or three degrees of freedom. The stiffness of these elements can be determined from the mechanical properties of the anisotropic solid to be represented by the DEM. Similar approach using other version of truss-like discrete element model could be consulted in Krajcinovic (1996) and Rinaldi (2011) among others papers.

Finally note that the so-called truss elements simply serve to visualize the direction of forces between two nodal masses, and are thus useful, principally for engineers, but do not exist physically (the truss elements are massless). Complete "failure" of an element simply implies that there is no force acting between the corresponding interconnected nodes and it does not mean that there is "fracture", unless all truss elements that cross a measurable surface are broken.

The version of Lattice Models used in the present paper, referred herein as DEM, was proposed by Riera (1984) to determine the dynamic response of plates and shells under impact loading when failure occurs primarily by shear or tension, which is generally the case in concrete structures. The DEM has been successfully used to solve structural dynamics problems, such as shells subjected to impulsive loading (Riera J.D., Iturrioz I 1998), the study of the scale effect in concrete, Rios and Riera 2004, and in rock dowels, Miguel et al 2008,. The computation of fracture parameters in static and dynamic problems Kostesky 2012, and recently in to simulate an Acoustical Emission in concrete body in Iturrioz et al (2013a.).

Notice that the introduction of heterogeneity nature in the model if we like simulate quasi-fragile material is crucial to obtain coherent results in the simulation of damage process.

In the present paper we explore the different ways to introduce this heterogeneity in the model. Two sources to include the heterogeneities are showed, that are: Considering the parameters that govern the bilinear constitutive law of the element as a random fields with specific probability function and correlation length. And introducing imperfection in the coordinates of the mesh lattice. The introduction of these two source of heterogeneities have allowed that with the same set of parameters and the simple bilinear constitutive law for the bars, capture the behavior up to rupture models with tension and compression solicitation.

When this model is used to represent dominant tensile solicitation the results are mainly influenced with the random field that govern the bilinear constitutive law.

If the model is submitted predominately in compression the results are mainly influenced with the level of imperfection, this characteristic is make use to facilitate the calibration of the model.

2 The Discrete Element Method in Fracture Problems

The Discrete Element Method employed in the present paper is based on the representation of a solid by means of a cubic arrangement of elements able to carry only axial loads. The discrete elements representation of the orthotropic continuum was adopted to solve structural dynamics problems by means of explicit direct numerical integration of the equations of motion, assuming the mass lumped at the nodes. Each node has three degrees of freedom, corresponding to the nodal displacements in the three orthogonal coordinate directions. In Fig. 1 a and b illustrate the basic bar arrange used in this approach.

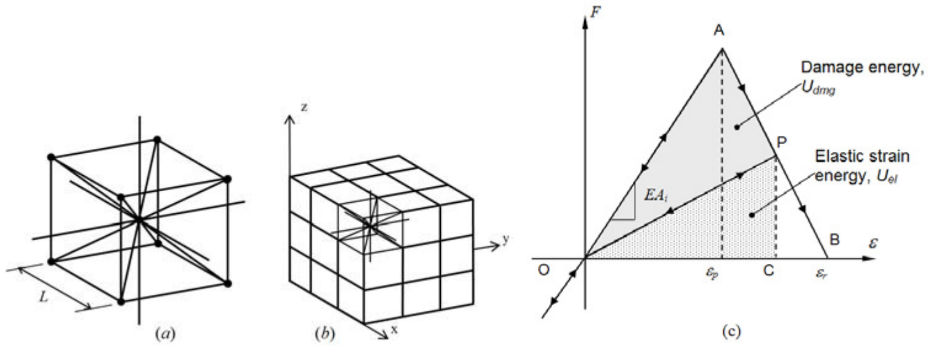


Figure 1:(a) basic DEM cubic module, (b) generation of a prismatic body,(c) Triangular constitutive law adopted for DEM uni-axial elements.

The equations that relate the properties of the elements to the elastic constants of an isotropic medium are:

$$\delta = \frac{9\nu}{4-8\nu} \ , \ EA_n = EL^2 \frac{(9+8\delta)}{2(9+12\delta)} \ , \ EA_d = \frac{2\sqrt{3}}{3} A_n \qquad (1)$$

in which E and ν denote Young's modulus and Poisson's ratio, respectively, while A_n and A_d represent the areas of normal and diagonal elements. The resulting equations of motion may be written in the well-known form:

$$\mathbf{M}\ddot{\vec{x}} + \mathbf{C}\dot{\vec{x}} + \vec{F}_r(t) - \vec{P}(t) = 0 \qquad (2)$$

in which $\vec{x}$ represents the vector of generalized nodal displacements, ***M*** the diagonal mass matrix, ***C*** the damping matrix, also assumed diagonal, $\vec{F}_r(t)$ the vector of internal forces acting on the nodal masses and $\vec{P}(t)$ the vector of external forces. Obviously, if ***M*** and ***C*** are diagonal, Eq. (2) is not coupled. Then the explicit central finite differences scheme may be used to integrate Eq. (2) in the time domain. Since the nodal coordinates are updated at every time step, large displacements can be accounted for in a natural and efficient manner.

2.1 Non-Linear Constitutive Model for Material Damage

The softening law for quasi-fragile materials proposed by Hilleborg (1971) was adopted to handle the behavior of quasi-fragile materials by means of the triangular elemental constitutive relationship (ECR) for the DEM bars presented in Fig. 2, which allows accounting for the irreversible effects of crack nucleation and propagation. The area under the force *vs.* strain curve (the area of the triangle OAB in Fig. 1.c) it is linked with the energy density necessary to fracture the influence area of the element. Thus, for a given point P on the force *vs.* strain curve, the area of the triangle OPC is linked with the reversible elastic energy density stored in the element, while the area of the triangle OAP is proportional to the energy density dissipated by damage. Once the damage energy density equals the fracture energy, the element fails and loses its load carrying capacity. On the other hand, in the case of compressive loads the material behavior is assumed linearly elastic. Thus, failure in compression is induced by indirect tension. The critical failure strain (ε_p) is defined as the largest strain attained by the element before the damage initiation (point A in Fig. 1.c). The relationship between ε_p and the specific fracture energy G_f is given in terms of Linear Elastic Fracture Mechanics as:

$$\varepsilon_p = R_f \sqrt{\frac{G_f}{E(1-\nu^2)}} \ (3) \ , \ R_f = \frac{1}{Y\sqrt{a}} \ (4),$$

in which R_f is the so-called failure factor, which may accounts for the presence of an intrinsic defect of size a. R_f may be expressed in terms of a. In the expression (3), Y is a dimensionless parameter that depends on both the specimen and crack geometry. The element loses its load carrying capacity when the limit strain ($\varepsilon_r = k_r\varepsilon_p$) is reached (Point B in Fig 1.c). Where k_r is a coefficient that link both characteristic strain in the micro constitutive law shown in Fig 1.c.
Could be useful here to define the fragility number s proposed by Carpinteri 1984 expressed as follows,

$$s=(G_f E)^{0.5}/\sigma_p D^{0.5} \text{ or } s= R_f^{-1} D^{-0.5} \text{ if } \sigma_p=E\varepsilon_p \qquad (5)$$

Where D is the characteristic size of the structure, σ_p the critical stress. This number have the property to characterize the ductility of the body. In the context of DEM if we have two examples with different sizes, that is, different D values and different material properties, (G_f, E and ε_p) but both cases have the same fragility number s, the both global response will have the same shape. More exhaustive explanation of this version of the lattice model could be found in the references indicated in the introduction of the present paper.
The influence of the parameters that govern the different sources of heterogeneities are:
a) The random distribution of constitutive law parameters of DEM: several works were done using this approach , see for example Miguel et al (2008) , where the random properties of the material defining the toughness G_f as a random field with a Type III (Weibull) extreme value distribution, given by Eq. (6), was used:

$$F\left(G_f\right) = 1 - \exp\left[-\left(G_f/\beta\right)^{\gamma}\right] \qquad (6)$$

in this expression β and γ are the scale and shape parameters, respectively. In earlier applications of the DEM, taking the size of the elements L equal to the correlation length of the random field of the material property of interest, say L_c, allowed assuming that simulated values were uncorrelated, thus simplifying the computational scheme. A simpler technique was employed by Puglia *et al.* (2010) to simulate the 3D random field that describes the toughness G_f, which is then independent of the discretization adopted in the DEM . That is, in this approach the correlation length of the random field L_c it isn't linked with the level of discretization L .

b) Perturbation of the DEM mesh coordinates: In the present formulation of the Discrete Element Method, solids are represented by means of a cubic arrangement of elements able to carry only axial loads, interconnected at nodal masses with three degrees of freedom. The initial elastic stiffness of the interconnecting elements is determined, for the cubic and other arrangements, in terms of the local elastic properties of an orthotropic solid, which may thus be non-homogeneous, by means of available sets of equations. The introduction of small perturbations, generated by small initial displacements of nodal points, should result in also small changes in the stiffness of the elements, which will tend to zero as the initial nodal displacements vanish. Hence, it is herein assumed that the stiffness coefficients of the DEM model remain unaltered by small perturbations of the mesh.

3 Analysis of Cubes Subjected to Uniaxial Load State

In the present section we present a set cubes of 0.15x0.15m submitted to uniaxial compression and tension test, we consider four kind of material that represent cubes with different fragility numbers (s=0.1, 0.2, 0.4 and 0.8).
In the Table 1 the four materials properties are shown. The heterogeneity is introduced considering that G_f , the parameter that rule the material toughness, is a random field with a Weibull density of probability with mean value (μ_{Gf}), variation coefficient (C_{Gf}) and correlation length L_c, defined as input parameters. The other source of randomness introduced is the mesh imperfection with a statistical Normal distribution and mean value equal to zero. Its effect depend on the level of discretization. In the present case the parameters that characterized the random nature of the model, were arbitrated. Similar values were used in previous studied Iturrioz et al (2013b) to simulate concrete material. The input parameters used are shown in Table 1. Notice that consider C_{Gf} in the simulation it is greater than the coefficient of variation that correspond to equivalent solid. Because in each solid cube of size L we have 14 bars, simulation studied show that C_{Gf} is approximately twice and half greater than a variation coefficient of equivalent solid. With this parameters it is possible determine that for s= 0.1, 0.2, 0.4, 0.8; $\mu(\varepsilon_p)$= 0.17x10^{-4}, 0.35x10^{-4}, 0.7x10^{-4}, 1.4 x10^{-4} and k_r = 256, 640, 160, 40 . The model was discretized with 40x40x40 cubic modules. The

discretization time used was Δt=0.5x10-6sec, that is lower than Δt_{min} given by the expression $L_{min}/(E/\rho)^{0.5}$, where L_{min} is the minimum bar length in all the model. The excitation over the models was applied as prescribed constant velocity displacement applied in the up of the cubes, taking into account that the velocity of displacement prescribed not produces considerable inertial effect in the simulations. Detail of the boundary conditions applied are presented directly over the final configurations in Fig.3 b for the compression test and in Fig. 3 d in the tensile test.

Table 1: Parameters adopted in the DEM cubes.

μ_{Gf}	C_p	E	C_{Gf}	ρ	ν	s=0.1	s=0.2	s=0.4	s=0.8	L	L_c
150N/m	2.0%	35Gpa	50%	2400K/m^3	0.25	R_{fc} =0.26	R_{fc} =0.52	R_{fc} =1.04	R_{fc} =2.08	0.0037m	L

3.1 Results Discussion

In the Fig. 2a, the response in terms of global stress vs. strain for the uniaxial compression and tension test, are presented for four materials tested with fragility number s=0.1 to 0.8 with parameters shown in Table 1.
Notice that the results were normalized respect the maximum stress (σ^* =7.376MPa) and the correspondent strain value (ε^* =6.141x10-4) for tensile test over the material characterized by s=0.8 body test.
Notice that, in the response of the cubes to the tensile test (Fig. 2.a), for s=0.4 and 0.8 the shape of the global response is characterized by a abrupt collapse. Nowadays it is possible in laboratory carry out tested with a active control of displacement to capture the instability branches of the response Li et al 1993, among others describing experimental techniques where these kind of control is carried out. In the simulation presented here, the velocity applied in the boundary remain constant in all the simulations. The simulation of test with control of stress with the aim to capture the instability branches in tension and compression test will be focus of future work. On the other hand in the case of s=0.1. 0.2 a stable behavior happened in all the damaged process.
In the results of compression test for the different s values the shape of the curves are similars, only change the max stress as shown in Fig 2.b, also notice, in the same Figs, that this curves have physical sense up to the slash limit line, after this point a sliding among the fissures happen in the real material and this damage mechanism it isn't account for in our model. This limit happened in general a bit after the maximum stress was reached.

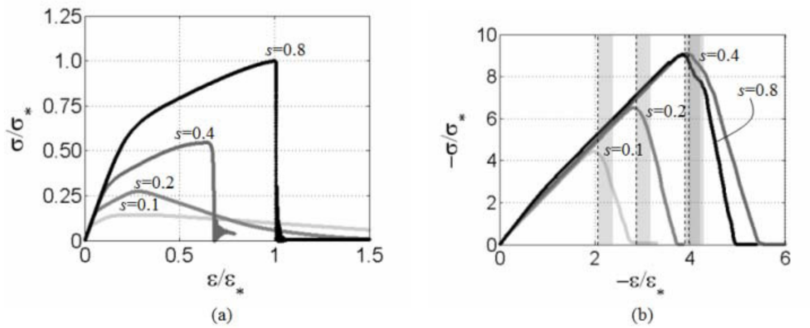
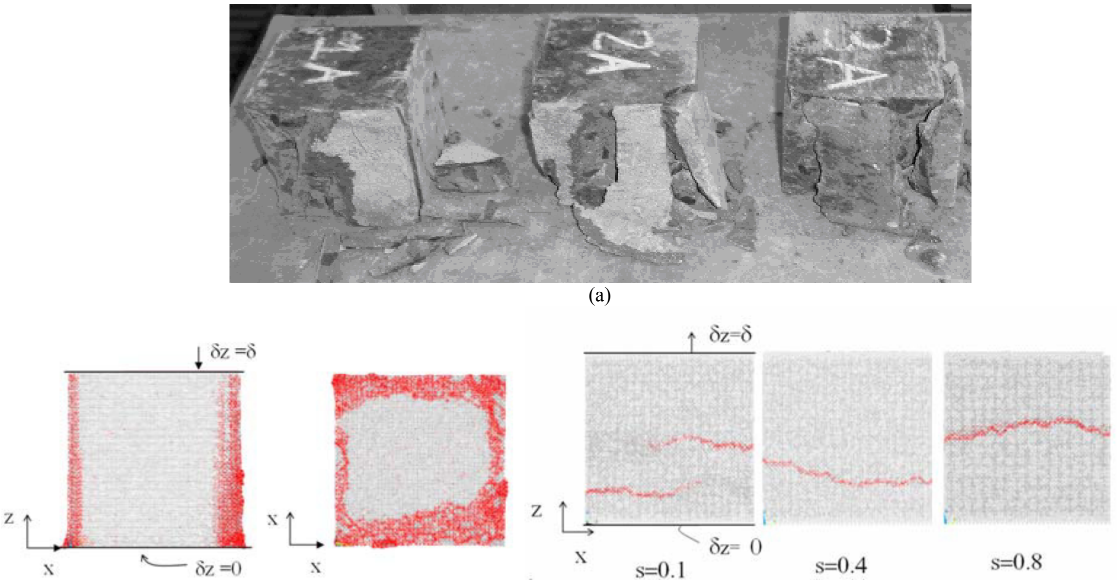


Figure 2: Global response in terms of global stress vs global strain for different s values. (a) In the uniaxial tensile test, (b) In the uniaxial compression test. (σ* =7.376MPa), (ε* =6.141x10-4).

In Fig. 3 b notice two views of the configuration for the case s=0.8 when the maximum strength is reach. It is possible appreciate that, in this case, the rupture begin in the lateral surface of the cube model. In Fig. 3.a we present a typical final configurations in concrete cubes with s similar to 0.8 where the same pattern of rupture appear. The pattern of rupture for other values of s in compression are similars.
In Fig. 3c we present the final configuration of the DEM simulation for the uniaxial tensile test for the cases s=0.1, 0.4,and 0.8. In the first case appear two macro fissures, and a high level of damage appears in the front of the main cracks. In the other cases (s=0.4 and 0.8), only one main fissure appear, and the final configuration not present apparently differences among them.
The results shown that this simple approach let us simulate with the same set of parameters the behavior during all

the damage process for the tension test, and up to reach the maximum stress in the compression test.



(a)

(b) (c)

Figure 3: (a) A typical experimental configuration for quase-fragile material. (a)Typical final configurations for concrete cube specimens submitted at compression test. (b) Final configuration obtained with DEM for s=0.8. (c) The final configurations for simple tensile test. For the cases s=0.1,s=0.4,s=0.8. In red are shown the bars that exhausted its strength in dark gray the bars with damage, and in light gray the undamaged bars.

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